

NUCLEON TENSOR CHARGES IN THE CHIRAL QUARK-SOLITON MODEL ¹

HYUN-CHUL KIM, MAXIM V. POLYAKOV ² and KLAUS GOEKE

*Institute for Theoretical Physics II,
P.O. Box 102148, Ruhr-University Bochum,
D-44780 Bochum, Germany*

Abstract

We investigate the singlet $g_T^{(0)}$ and isovector $g_T^{(3)}$ tensor charges of the nucleon, which are deeply related to the first moment of the leading twist transversity quark distribution $h_1(x)$, in the chiral quark-soliton model. With rotational $O(1/N_c)$ corrections taken into account, we obtain $g_T^{(0)} = 0.69$ and $g_T^{(3)} = 1.45$ at a low normalization point of several hundreds MeV. Within the same approximation and parameters the model yields $g_A^{(0)} = 0.36$, $g_A^{(3)} = 1.21$ for axial charges and correct octet-decuplet mass splitting. We show how the chiral quark-soliton model interpolates between the nonrelativistic quark model and the Skyrme model.

The nucleon tensor charges are defined as a nucleon forward matrix element [1]

$$\langle N | \bar{\psi}_f \sigma_{\mu\nu} \psi_f | N \rangle = g_T^f \bar{U} \sigma_{\mu\nu} U, \quad (1)$$

where $U(p)$ is a standard Dirac spinor and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ in notation of Bjorken and Drell [2]. The tensor charges, as shown by Jaffe and Ji [1], are related to the first moment of the transversity quark distribution $h_1(x)$:

$$\int_0^1 dx (h_1(x) - \bar{h}_1(x)) = g_T^f, \quad (2)$$

where f is a flavor index ($f = u, d, s, \dots$).

Our aim is to calculate the tensor charges (1) in the chiral quark-soliton model (χ QSM, often called the semibosonized Nambu—Jona-Lasinio model) at a low normalization point of several hundreds MeV.

The χ QSM has been successful in reproducing the static properties of the baryons such as the octet-decuplet mass splitting [3, 4], axial charges [5, 6, 7] and electro.m. form factors [8, 9]. The baryon in this model is regarded as a bound state of N_c quarks bound by a non-trivial chiral field configuration. Such a semiclassical picture of baryons can be justified in the $N_c \rightarrow \infty$ limit in line with more general arguments by Witten [10]. A remarkable virtue of χ QSM is that the model interpolates between the nonrelativistic quark model (NRQM) and the Skyrme model [11]. In particular, due to such an interplay, it enables us to examine the dynamical difference between the axial and tensor charges of the nucleon.

¹Talk given at 7th International Conference on the Structure of Baryons, Santa Fe, New Mexico, 3-7 Oct 1995.

²On leave of absence from Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188350, Russia

In order to calculate the tensor charges given by eq.(1) we employ the effective QCD partition function following from the instanton picture of QCD in the limit of low momenta. It is given by a functional integral over pseudoscalar and quark fields [12]:

$$\mathcal{Z} = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{D}\pi^A \exp \left(i \int d^4x \bar{\Psi} iD \Psi \right), \quad (3)$$

where iD denotes the Dirac differential operator

$$iD = (-i\vec{\not{D}} + M e^{i\pi^A \tau^A \gamma_5}), \quad (4)$$

and M is the dynamical quark mass which arises as a result of the spontaneous chiral symmetry breaking and is momentum-dependent. The momentum dependence of M introduces the natural ultra-violet cut-off (inverse average instanton size $1/\rho \sim 600$ MeV) [12] for the theory given by eq. (3) and simultaneously brings a renormalization scale to the model.

One can relate the hadronic matrix element eq. (1) to a correlation function:

$$\langle 0 | J_B(\vec{x}, T) \bar{\psi} \sigma_{\mu\nu} \tau^a \psi J_B^\dagger(\vec{y}, 0) | 0 \rangle \quad (5)$$

at large Euclidean time T with baryon current J_B constructed from quark fields and having nucleon quantum numbers. The correlation function (5) can be calculated in the effective chiral quark model defined by eq.(3) using $1/N_c$ expansion. The related technique can be found in [3, 8]. Here we give a result for the tensor charges to the next to leading order of the $1/N_c$ expansion:

$$g_T^{(0)} = \frac{\alpha}{I}, \quad g_T^{(3)} = \beta + \frac{\delta}{I}, \quad (6)$$

where α, β, δ and $I \sim N_c$ can be found in ref.[13].

The χ QSM interpolates between NRQM and the Skyrme one, *i.e.* in the limit of small soliton size it reproduces the results of NRQM, whereas in the opposite limit of large soliton size it mimics the Skyrme model. In the limit of large soliton size (large constituent quark mass), one can easily find [13] that $\alpha \sim (MR_0)^2$, $I \sim (MR_0)^3$ and $\beta, \delta \sim MR_0$. Therefore, the ratio of the tensor charges $g_T^{(0)}/g_T^{(3)} \sim 1/(MR_0)^2$ is sizably reduced in the limit of large soliton size, while the analogous analysis of the axial charges [6, 11] gives even much stronger suppression in the ratio $g_A^{(0)}/g_A^{(3)} \sim 1/(MR_0)^6$. This observation of the different behaviors between the axial and tensor charges leads to a conclusion that the tensor charges might deviate from axial ones remarkably. In the limit of $MR_0 \rightarrow 0$, χ QSM corresponds to NRQM and yields: $g_T^{(0)} = g_A^{(0)} = 1$, $g_T^{(3)} = g_A^{(3)} = (N_c + 2)/3$ (derivation for axial charges see ref.[11])³.

In figure 1 we show the dependence of the tensor and axial charges on the soliton size. We see that axial and tensor charges starting from the same values of $(N_c + 2)/3 \approx 1.67$ for the isovector case and 1 for the singlet one at small soliton size have qualitatively different behavior for larger MR_0 — the dependence of the tensor charges on soliton size is weaker than corresponding dependence of the axial charges. This qualitative difference is in accordance with the asymptotics of the charges in large soliton size considered above.

³Note that it is of great importance to take into account the rotational $1/N_c$ corrections (δ contribution in eq. (6)) to derive this result in $O(N_c^0)$ order.

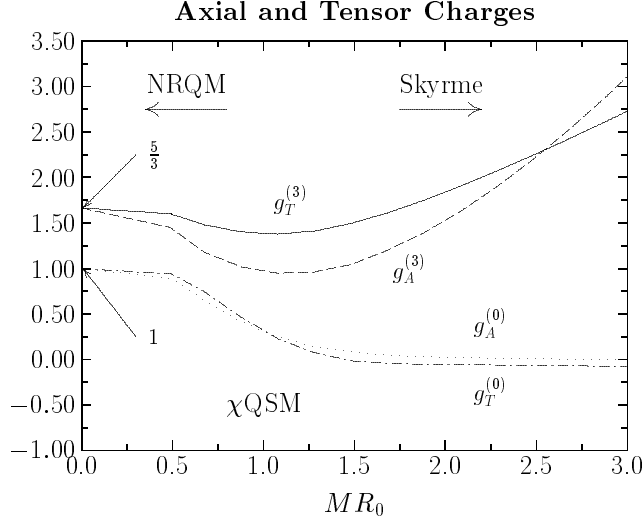


Fig. 1: The dependence of the axial and tensor charges on the soliton size. The solid curve represents the $g_T^{(3)}$, while the dashed curve draws the $g_A^{(3)}$. The dot-dashed curve denotes the $g_T^{(0)}$, whereas the dotted curve is for the $g_A^{(0)}$. The small arrows stand for the values of $g_T^{(3)} = g_A^{(3)} = 5/3$ and $g_T^{(0)} = g_A^{(0)} = 1$ in NRQM, respectively. The large arrows denote NRQM and Skyrme limit of the present model.

We have calculated the tensor charges for $M = 420$ MeV. At this mass the model reproduces with good accuracy many nucleon observables – octet-decuplet mass splitting [4], isospin splittings in baryon octet and decuplet [14], singlet axial charge [6, 7], magnetic moments, isovector axial charge [5] and electromagnetic form factors [8, 9]. Using accurate Kahana–Ripka method [15] for diagonalization of the Dirac operator, we got:

$$g_T^{(3)} \approx 1.45, \quad g_T^{(0)} \approx 0.69. \quad (7)$$

We find that the obtained results are close to those in the bag model [1] and consistent with QCD sum rule calculations of refs. [16, 17]. Using the same technique and parameters of the model one obtains the following values of the axial charges:

$$g_A^{(3)} \approx 1.21, \quad g_A^{(0)} \approx 0.36. \quad (8)$$

It is worth noting that a dependence of the tensor charges on the normalization point is rather weak:

$$g_T^{(f)}(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{4}{27}} g_T^{(f)}(\mu_0), \quad (9)$$

as $\mu \rightarrow \infty$ the tensor charges slowly vanish. One can use this equation to recalculate the tensor charges at higher normalization points using the values of tensor charges (7) at low normalization point. Due to the weak dependence of the tensor charges on the normalization point, we do not need to know precisely the value of the initial normalization point.

We would like to thank Chr. Christov and T. Watabe for helpful discussions and comments. This work has partly been supported by the BMFT, the DFG and the COSY–Project (Jülich). The work of M.P. is supported in part by grant INTAS-93-0283.

References

- [1] R. Jaffe, X. Ji, Phys. Rev. Lett. **67** (1991) 552.
- [2] J.D. Bjorken and S.D. Drell, “*Relativistic Quantum Mechanics*”, McGraw–Hill, New York (1964).
- [3] D.Diakonov, V.Petrov and P.Pobylitsa, Nucl.Phys. **B306** (1988) 809.
- [4] A. Blotz *et al.*, Nucl.Phys. **A555** (1993) 765.
- [5] C.V. Christov *et al.*, Phys. Lett. **325B** (1994) 467.
- [6] A. Blotz, M.V. Polyakov, and K. Goeke, Phys. Lett. **302B** (1993) 151.
- [7] A. Blotz, M. Praszalowicz and K. Goeke, RUB-TPII-41/93 [hep-ph/9403314] (1993).
- [8] Ch. Christov, A.Z. Górski, K. Goeke and P.V. Pobylitsa, Nucl. Phys. **A592** (1995) 513.
- [9] H.-C. Kim, A. Blotz, M. Polyakov, and K. Goeke, RUB-TPII-7/95 [hep-ph/9504363] to appear in Phys. Rev. D (1995).
- [10] E.Witten, *Nucl.Phys.* **B223** (1983) 433.
- [11] M. Praszalowicz, A. Blotz and K. Goeke, Phys. Lett. **354 B** (1995) 415.
- [12] D.Dyakonov and V.Petrov, *Nucl.Phys.* **B272** (1986) 457; preprint LNPI-1153 (1986), published in: *Hadron matter under extreme conditions*, Kiev (1986) p.192.
- [13] H.-C. Kim, M. V. Polyakov and K. Goeke, RUB-TPII-26/95, to appear in Phys. Rev. D. (1995).
- [14] M. Praszalowicz, A. Blotz, and K. Goeke, Phys. Rev. **47** (1993) 1127.
- [15] S. Kahana and G. Ripka, Nucl. Phys. **A429** (1984) 462.
- [16] H. He, X. Ji, *The Nucleon’s Tensor Charge*, MIT-CTP-2380 [hep-ph/9412235] (1994)
- [17] B.L. Ioffe, A. Khodjamirian, Phys. Rev. **D51** (1995) 3373-3380.